Quantum simulation of partially distinguishable boson sampling

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1 Introduction

Since quantum computers were first proposed, there have been speedups shown for a number of problems [1], but which require large fault-tolerant quantum computers. This has led to the search for a near-term experiment that is hard to simulate [2]. Numerous proposals exist, with perhaps the best known being Boson Sampling [3]. It was famously shown that classically sampling from n indistinguishable photons output from an m-mode linear optical interferometer is hard, modulo several conjectures. This has led to significant interest and experimental demonstrations [4, 5].

However, outperforming classical simulation is difficult [6, 7, 8]. Any advantage will also need to consider challenges such as distinguishability, where there has been significant interest [9, 10, 11].

Here we consider sampling from n partially distinguishable single bosons interacting on an m-mode interferometer from a quantum simulation perspective. We observe that ideal Boson Sampling is equivalent to sampling from the symmetric representation of the unitary group, and that partial distinguishability generalises this. The quantum Schur transform [12] can therefore be used to give a quantum algorithm for sampling, regardless of distinguishability. This shows how nonideal linear optics can be viewed as a quantum computation. Full details, including proofs, can be found in [13].

2 Preliminaries

We start by defining the ideal probability distribution of indistinguishable single bosons interacting on a linear interferometer. We'll denote this bosonic sampling, as it's a bit more general than Boson Sampling. The input is $U \in \mathrm{U}(m)$, an $m \times m$ unitary matrix which describes an m-mode linear interferometer, and $S = (S_1, S_2, \ldots, S_m)$ with $\sum_{i=1}^{m} S_i = n$, an ordered list of integers that corresponds to an n-boson, m-mode occupation describing the input state with S_i bosons in mode i. The probability of sampling the output state S' given U and S is related to the permanent of a complex matrix constructed from U. It was proven by Aaronson and Arkhipov that when $S = (1^n, 0^{n-m})$, efficient classical simulation would imply collapse of the Polynomial Hierarchy [3].

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Our algorithm can be understood from the perspective of the representation theory of the unitary group $\mathrm{U}(m)$. The irreducible representations, or irreps, are intimately related to those of the symmetric group S_n. Irreps of both of these groups are indexed by labels λ.

The actions of the symmetric and unitary groups on a state $|\Psi\rangle \in (\mathbb{C}^m)^{\otimes n}$ are as follows. For a permutation $\sigma \in \mathrm{S}_n$, the action permutes the qudits. For a unitary matrix $U \in \mathrm{U}(m)$, the action is the n-fold tensor product $U^{\otimes n}$. Schur-Weyl duality shows that the Hilbert space of n m-dimensional qudits decomposes into irreducible subspaces for these groups [12].

There is a quantum circuit that implements the Schur-Weyl decomposition. Given a state $|\Psi\rangle \in (\mathbb{C}^m)^{\otimes n}$ in the computational basis, this circuit, which we denote W, performs the transformation

$$W|\Psi\rangle = \sum_{\lambda \vdash n} \sum_{q_\lambda} \sum_{p_\lambda} C^{\lambda}_{q_\lambda, p_\lambda} |\lambda\rangle |q_\lambda\rangle |p_\lambda\rangle, \tag{1}$$

where λ indexes the irrep, q_λ and p_λ index bases of irreps of $\mathrm{U}(m)$ and S_n respectively, and $C^{\lambda}_{q_\lambda, p_\lambda}$ is a coefficient. For example, the unitary action of $\mathrm{U}(m)$ in this basis is

$$U : |\lambda\rangle |q_\lambda\rangle |p\rangle \to |\lambda\rangle |U \cdot q_\lambda\rangle |p\rangle := |\lambda\rangle \left(\sum_{q'_\lambda} U^{\lambda}_{q_\lambda, q'_\lambda} |q'_\lambda\rangle \right) |p\rangle, \tag{2}$$

where U^λ is the irreducible unitary matrix corresponding to $U \in \mathrm{U}(m)$. It was proven that this circuit runs with accruacy δ in polynomial time in terms of n, m and $\log(\delta^{-1})$ [12].

3 Quantum circuits for bosonic sampling

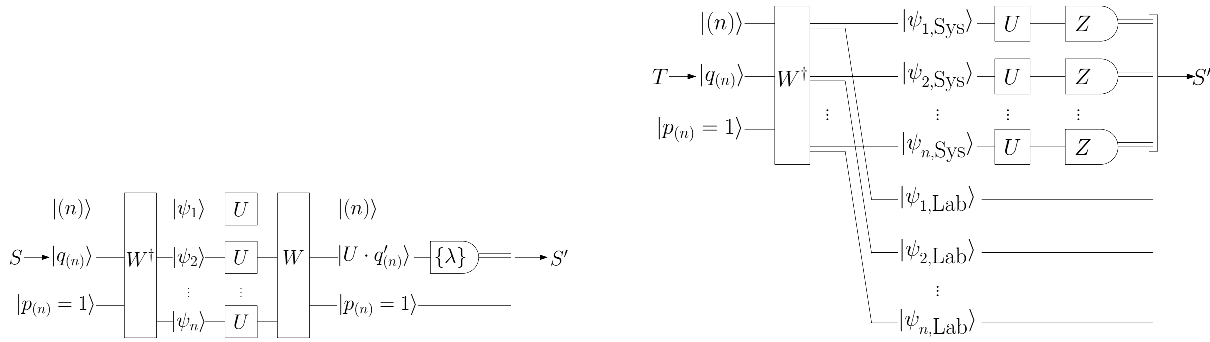
Here we describe quantum circuits for bosonic sampling. These have accuracy $\delta + \epsilon$ from approximating W and U, and run in polynomial time and space in terms of m, n, $\log(\delta^{-1})$ & $\log(\epsilon^{-1})$.

3.1 Ideal bosonic sampling

The goal for bosonic sampling with indistinguishable photons is to sample from the symmetric irrep of the unitary group, where the interferometer $U \in \mathrm{U}(m)$ acts on the symmetric subspace of $(\mathbb{C}^m)^{\otimes n}$. In order to construct symmetrised states from S, we use the Schur transform. The Schur circuit specifies irreps of $\mathrm{U}(m)$ in the Gelfand-Zeitlin (GZ) basis, so we need to map from these states to occupations. For the symmetric irrep, there is a simple one-to-one mapping [14], so we have an efficient way to identify an input occupation S with a GZ basis state $q_{(n)}$.

We can now see how a circuit for ideal bosonic sampling would work. Given input occupation S, we prepare the corresponding state $|q_{(n)}\rangle$. To use the inverse Schur transform, we append to this input state registers for the irrep $|(n)\rangle$ and symmetric group index $|p_{(n)}\rangle$. There is only one possible state for the $p_{(n)}$ register, as the fully symmetric irrep of the symmetric group is one dimensional, so $p_{(n)} = 1$. The inverse Schur transformation $W^\dagger$ takes this state to an n-qudit symmetric state $|\Psi_I\rangle \in (\mathbb{C}^m)^{\otimes n}$. In this space, we now need only apply the interferometer matrix U to each qudit in parallel as the circuit $U^{\otimes n}$. Finally, we apply the Schur transform again and measure the q-register to get a sample $q'_{(n)}$, from which we can easily compute an output occupation S'. This circuit is given in Fig. 1a.

It is worth noting that, instead of performing the second Schur transform, we can simply measure in the computational basis and write the outcome as an occupation. We use this in Sec. 3.2.



(a) A quantum circuit for idealised bosonic sampling.

(b) A quantum circuit for bosonic sampling with partially-distinguishable photons.

Figure 1: Quantum circuits for bosonic sampling. Here W denotes the quantum circuit for the Schur-Weyl duality [12], and U the interferometer. Note that in Fig. 1a, measurement is taken in the G-Z $\{\lambda\}$ basis, whereas in Fig. 1b, measurements are taken in the computational Z basis.

3.2 Arbitrarily distinguishable bosonic sampling

We now consider sampling from a distribution of partially distinguishable bosons. Distinguishability is modelled as correlation between the bosons' 'System' degrees of freedom, and new modes corresponding to 'Label' degrees of freedom. As every boson might be distinguishable, there must be n Label modes, giving a total of mn modes. We can think of the System and Label as spatial and, say, temporal modes, respectively. We assume that an interferometer $U \in \mathrm{U}(m)$ acts only on the System modes. Rather than receiving S as input in this model, we receive an $m \times n$ occupation T describing how many bosons are in System mode i and Label mode j. Arbitrarily distinguishable inputs can be constructed from this basis.

For distinguishable bosons, we still consider the symmetric irrep of the Unitary group. However, now we map onto the symmetric irrep of $\mathrm{U}(mn)$, since the total state must be symmetrised. It can be seen that tracing out the Label then leads to decoherence in the System register.

When we apply $W^\dagger$ to T, we get a symmetrised state $|\Psi\rangle \in (\mathbb{C}^m \otimes \mathbb{C}^n)^{\otimes n}$. Each System-Label qudit can be viewed as bipartite, with an m-dimensional qudit describing the System and an n-dimensional qudit describing the Label. We split the System-Label qudits into two registers, with the interferometer and measurement acting on only the System. This circuit is given in Fig. 1b.

4 Conclusion

The Schur transform lets us simulate bosonic sampling with arbitrary distinguishability. This shows that ideal bosonic linear interferometry is equivalent to a transversal quantum circuit with a symmetric input. Moreover, we can consider nonideal aspects of bosonics. A broad aim of future research is to better understand the link between complexity and distinguishability. Full details, including proofs, links to representation theory, bosonic sampling with photon loss, and multipartite entanglement of the System, can be found in [13]. No new data were created during this study.

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