A ZX re-writing system for surface code lattice surgery

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Surface codes with lattice surgery are currently the leading proposal for the large-scale quantum error correction needed for fault tolerant quantum computing. Whereas quantum computation is usually based on the application of unitary operators (or gates) alongside measurement, lattice surgery uses logical fusion and splitting operations that are inherently non-unitary. The fusion (merge) operation in particular takes two logical qubits as input, and outputs one logical qubit. This operation clearly involves an inherent loss of (quantum) information. Such operations are not primitives in the usual circuit representation of quantum gates, making the design and verification of lattice surgery patterns a hard task in the absence of better representational tools. Previous work has shown that the operations of lattice surgery map to collections of generators of the ZX calculus, leading to the idea that ZX should become the language of choice for lattice surgery procedures. In this work we give the formal semantics and re-write system for such collections of ZX diagrams that correspond to lattice surgery procedures. This has been achieved by first considering the spectrum of an operator, defined as a set of density operators or CPTP maps. Re-writes are then defined both individually between elements in a spectrum, and as spectra of the re-writes themselves. We finish by giving the conditions under which these rewrites are complete. The move towards a complete language for lattice surgery offers a coherent mechanism to reason about about surface codes. It also forms part of a new model of quantum computing based on probabilistic operations and the ZX calculus, termed Pauli Fusion. These results will broaden the tool-set available to all those working on the algorithms and foundations of computation as performed using quantum systems.

1 Introduction

Surface codes encode one logical qubit in multiple physical qubits for the purposes of error correction. Lattice surgery is a proposed model for working with these objects logically [8]. The basic operations correspond to taking lattices representing qubits and fusing/splitting them. This is manifestly non-unitary: quantum information is lost during the merge, when two qubits become one. As a consequence, merging is probabilistic. When two codes merge together one surface may be π out of phase with the other (i.e an opposite basis element) and so must be corrected.

The ZX calculus [2][3] is a language for expressing qubit quantum mechanics and has found success in this being complete for pure [10], Clifford [1], and Clifford+T quantum mechanics [9, 4]. Previous work has shown a close connection between lattice surgery operations and ZX generators [6]. In this present work we consider an extension of the ZX calculus to work directly with the probabilistic biproducts of lattice surgery and to re-write such diagrams. This question has been

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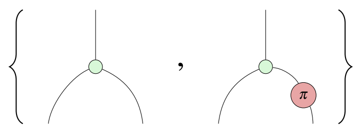
approached before in [7], but with mixed results with respect to designing an axiom and rewrite system due to there being numerous possible probabilistic phases. For lattice surgery, however, the only probabilistic elements are π phases. Such operations are represented by Pauli Fusion diagrams [5]. We present here axioms for Clifford operations in lattice surgery that are 'lifted' from standard ZX axioms. These axioms are complete, giving an intuitive and user-friendly design and verification language for lattice surgery.

This is an extended abstract for poster submission.

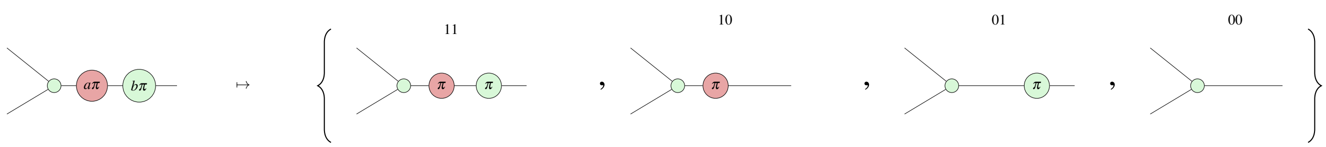
2 LS-PF: probabilistic ZX diagrams for lattice surgery

In [6] we see that lattice surgery procedures correspond to either single ZX diagrams or, in the particular case of the merge operations, one of two ZX diagrams:



We encapsulate this notion in LS-PF diagrams, which are diagrams with a possible π phase e.g.

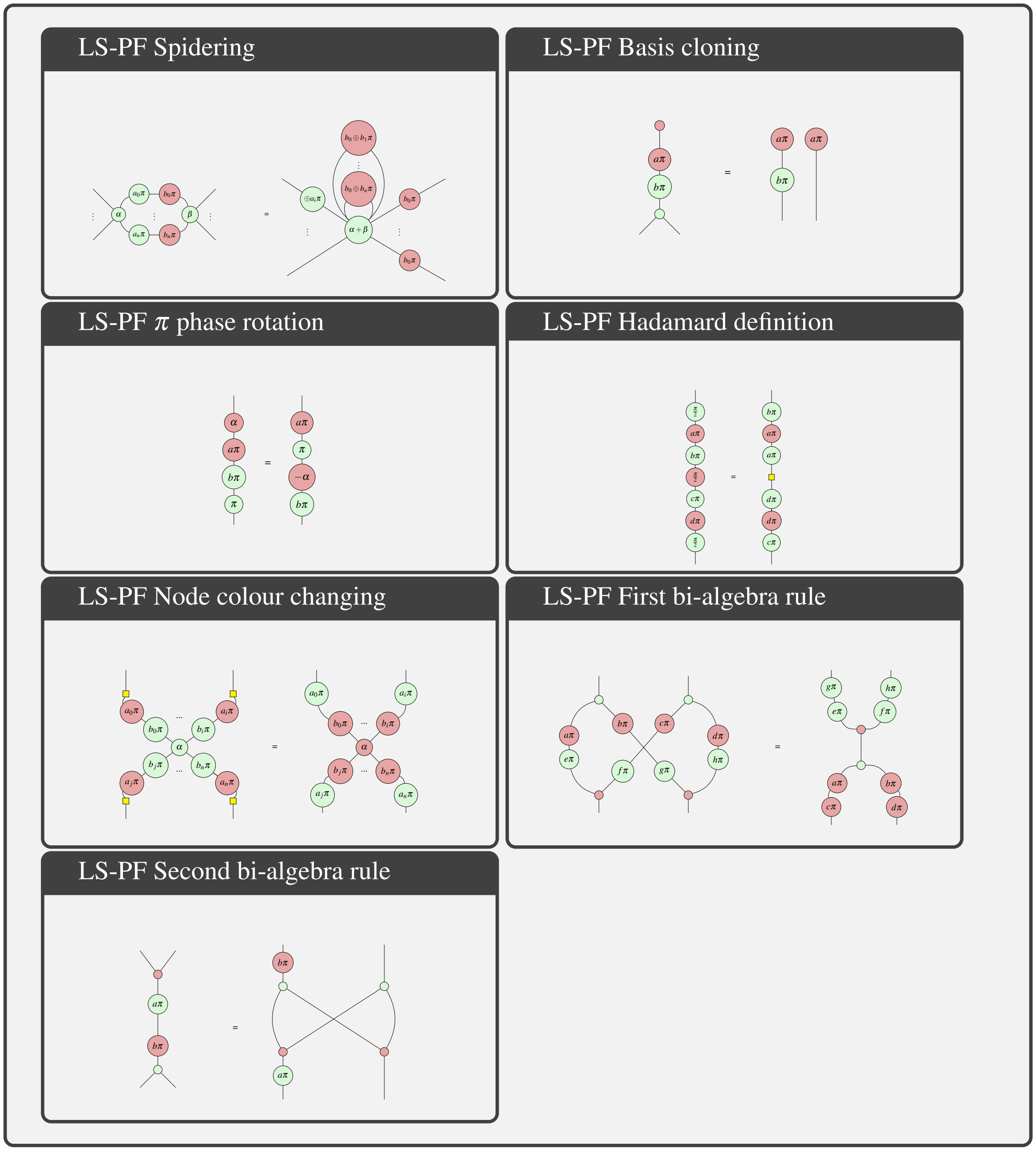
These diagrams naturally give rise to sets or *spectrums* of ZX diagrams that are composed of all the possible outcome diagrams which are obtained by mapping the probabilistic labels into $\mathbb{B}$. For example:



A diagram is a representation of CPTP maps, and its spectrum is of Kraus operators.

3 Re-writing LS-PF diagrams

The generalised axioms give a structure such that by assigning a binary number to a_i and b_i in all combinations one obtains all possible rewrites where there are 'complications' to the standard ZX re-write from a possible π element. In general there could be any countable number of a π-node of a particular colour interfering with a rewrite but as π-nodes of different colours can pass each other unaltered and π-nodes of the same colour either cancel or leave only one π phase, depending on there being an even or an odd number, one can see diagrammatically one need only consider two cases for each node colour: one node or none. With this in mind the LS-PF generalised Clifford ZX axioms are as follows:



We can prove that these axioms inherit the completeness of the Clifford ZX axioms to form the set of Clifford LS-PF axioms.

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