Anomalous weak values and contextuality: robustness, tightness, and imaginary parts

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Weak values are quantities accessed through quantum experiments involving weak measurements followed by post-selection. It has been shown that weak values whose *real part* is 'anomalous', i.e. lies outside the eigenvalue range of the corresponding operator, defy classical explanation in the sense of requiring contextuality [M. F. Pusey, Phys. Rev. Lett. **113**, 200401, arXiv:1409.1535]. Here we elaborate on and extend that result in several directions. Firstly, the original theorem requires the observation of certain perfect correlations that can never be realised in any actual experiment. Hence, we provide new theorems suitable to experimental verification. Secondly, the original theorem connects the anomaly to contextuality *only* in the presence of a whole set of extra operational constraints. Here we clarify the debate surrounding anomalous weak values by showing that all these extra conditions are indeed necessary – if any one of them is dropped, the anomaly can be reproduced classically. Thirdly, whereas the original result required the *real part* of the weak value to be anomalous, we also give a version for any weak value with nonzero imaginary part. Finally, we show that similar results hold if the weak measurements is performed through qubit pointers, rather than the traditional continuous system. In summary, we provide tight inequalities for witnessing nonclassicality using experimentally realistic measurements of *any* anomalous weak value, and clarify what ingredients of the quantum experiment must be missing in any classical model that can reproduce the anomaly.

This submission is based on arXiv:1812.06940.

Weak measurements [1] are a class of minimally disturbing quantum measurements whose practical as well as foundational relevance is currently being investigated [2]. A weak measurement of an observable O can be realized by weakly coupling a quantum system to a one-dimensional pointer device via a von Neumann-type interaction $\propto O \otimes P$, with P the momentum of the pointer, so that a small amount of information is imprinted in the pointer at the cost of a small disturbance on the system.

Pivotal to any attempt to establish the presence of nonclassical effects in a given experiment is the formulation of a rigorous no-go theorem based on a precise and operational notion of nonclassicality. It has long been argued that the average final position of the pointer – conditioned upon a successful postselection performed after the weak measurement – is a witness to nonclassicality [1]; in the quantum formalism this quantity is related to the (real part of the) *weak value*, which is ${}_\phi\langle O\rangle_\psi := \langle\phi|O|\psi\rangle \,/\, \langle\phi|\psi\rangle$, where O is the observable being weakly measured, $|\psi\rangle$ is the initial preparation and $|\phi\rangle$ is the post-selection. A long-standing debate ensued between those supporting the thesis that these experiments are indeed probing truly quantum effects and those arguing that they can be easily understood from classical statistics [3–8].

Recently, a precise no-go theorem was established [9]. The theorem proves that *anomalous weak values* (AWV), i.e. ${}_\phi\langle O\rangle_\psi$ taking values outside the spectrum of O, are associated to operational statistics defying any noncontextual explanation in the generalized sense introduced in Ref. [10]. Nevertheless, the theorem of Ref. [9] leaves several questions open:

1. First of all, it assumes an *exactly* projective post-selection $|\phi\rangle$, which makes any experimental test necessarily inconclusive; in fact, any degree of noise makes the no-go theorem inapplicable. Does the nonclassicality of AWV survive real-world conditions?

2. Second, both Ref. [9] and the noise-robust theorems we obtain here require a set of extra operational conditions to be satisfied. Are these truly necessary?

3. Third, the theorem only refers to the *real part* of the weak value. Is a nonzero value of the imaginary part of the weak value also non-classical?

4. Fourth, the relation between AWV and contextuality holds for a measurement with a continuum of outcomes. Can it be extended to discrete systems, such as an experiment involving only a single qubit pointer, or a coarse graining of the standard weak value experiment? This is also relevant because the infinitely many operational constraints required for the original theorem to hold cannot be tested by finite means.

5. Finally, the theorem identifies a single noncontextuality inequality which is violated in the presence of AWV. However, is the inequality unique and is it tight?

Our investigation answers all these questions:

1. We provide two new proofs of contextuality from AWV that are robust to noise. The two new proofs are complementary, each requiring the satisfaction of a different set of operational constraints together with the observation of the AWV. These results show that, at the price of extending the set of operational tests required, the relation between AWV and nonclassicality extends beyond the ideal, noiseless case. We also discuss the significance of these results for current experimental tests.

2. We show that dropping *any one* of the extra operational conditions required in our theorems allows to reproduce the AWV within a classical model. This illuminates the debate around "quantumness" of AWV (*e.g.*, [5–7]), since it rigorously shows that it is only in the presence of *all* the operational facts listed in our theorems that AWV defy a classical explanation.

3. The imaginary part of the weak value admits its own contextuality theorem. Hence, *any* AWV can be related to contextuality. We clarify why this is not in contradiction with recent studies [11, 12] suggesting that imaginary weak values admit a classical model.

4. The contextuality of AWV has nothing to do with continuous measurements, and extends to discrete pointers as well. This makes the experiment suited for conclusive experimental verification, since in this case only a finite set of operational tests are required.

5. The noncontextual bound in Ref. [9] is not tight, but we provide an improved, tight version and discuss its uniqueness.

A detailed technical account of these results can be found in arXiv:1812.06940.

All in all, our results show that contextuality captures what is nonclassical about anomalous weak values in a way that is experimentally relevant and wide-ranging. In particular, the postselection need not be a perfect projective measurement, the pointer need not be a continuous-variable system, and if there is an imaginary part to the weak value then the real part need not be anomalous. On the other hand, we show through explicit noncontextual models that if any of the operational equivalences we use are absent, a classical explanation is possible.

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