A comonadic view of simulation and quantum resources

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We study simulation and quantum resources in the setting of the sheaf-theoretic approach to contextuality and non-locality. Resources are viewed behaviourally, as empirical models. In earlier work a notion of a morphism for these empirical models was proposed and studied. We generalize and simplify the earlier approach, by starting with a very simple notion of morphism, and then extending it to a more useful one by passing to a co-Kleisli category with respect to a comonad of measurement protocols. We show that these morphisms capture notions of simulation between empirical models obtained via "free" operations in a resource theory of contextuality, including the type of classical control used in measurement-based quantum computation schemes.

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A key objective in the field of quantum information and computation is to understand the advantage which can be gained in information processing tasks by the use of quantum resources. While a range of examples have been studied, to date a systematic understanding of quantum advantage is lacking.

One approach to achieving such a general understanding is through *resource theories* [9, 1], in which one considers a set of operations by which one system can be transformed into another. In particular, one considers "free operations", which can be performed without consuming any additional resources of the kind in question. If resource B can be constructed from A using only free operations, then we say that A is convertible to B, or B is reducible to A. This point of view is studied in some generality in [7, 8].

Another natural approach, which is familiar in computation theory, is to consider a notion of *simulation*; one asks if the behaviour of B can be produced by some protocol using A as a resource.

Both these points of view can be considered in relation to quantum advantage. Our focus in this paper is on quantum resources which take the form of *non-local*, or more generally *contextual*, correlations. Contextuality is one of the key signatures of non-classicality in quantum mechanics [12, 5], and has been shown to be a necessary ingredient for quantum advantage in a range of information processing tasks [13, 10, 6, 1]. In previous work [1], we showed how this advantage could be quantified in terms of the *contextual fraction*, and we also introduced a range of free operations, which were shown to have the required property of being non-increasing with respect to the contextual fraction. Thus this work provided some of the basic ingredients for a resource theory of quantum advantage, with contextuality as the resource.

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In [11], one of the present authors introduced a notion of simulation between (possibly contextual) behaviours, as morphisms between empirical models, in the setting of the "sheaf-theoretic" approach to contextuality introduced in [2]. This established a basis for a simulation-based approach to comparing resources.

In this paper, we bring these two approaches together.

- On the simulation side, we enhance the treatment given in [11] by introducing a *measurement protocols* construction on empirical models. Measurement protocols were first introduced in a different setting in [3]. This construction captures the intuitive notion, widely used in an informal fashion in concrete results in quantum information (e.g. [4]), of using a "box" or device by performing some measurement on it, and then, depending on the outcome, choosing some further measurements to perform. This form of adaptive behaviour also plays a crucial role in measurement-based quantum computing [14].

  We show that this construction yields a comonad on the category of empirical models, and hence we are able to describe a very general notion of simulation of B by A in terms of co-Kleisli maps from A to B.

- We consider the algebraic operations previously introduced in [1] and introduce a new operation allowing a conditional measurement, a one-step version of adaptivity. We present an equational theory for these operations and use this to obtain normal forms for resource expressions.

- Using these normal forms, we obtain one of our main results: we show that the algebraic notion of convertibility coincides with the existence of a simulation morphism.

- We also prove some further results, including a form of no-cloning theorem at the abstract level of simulations.

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